

High-fidelity manipulation of a Bose-Einstein condensate using Bragg interactions

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The use of off-resonant standing light waves to manipulate ultracold atoms is investigated. Previous work has illustrated that optical pulses can provide efficient beam-splitting and reflection operations for atomic wave packets. The performance of these operations is characterized experimentally using Bose-Einstein condensates confined in a weak magnetic trap. Fidelities of 0.99 for beam splitting and 0.98 for reflection are observed, and splitting operations of up to third order are achieved. The dependence of the operations on light intensity and atomic velocity is measured and found to agree well with theoretical estimates.

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Optical control of atomic motion has developed into a fertile field over the past few decades. One important tool is the off-resonant standing wave laser beam [1, 2]. Atoms in such a beam experience a spatially oscillating potential that can act as a diffraction grating for the atomic wave function in the same way that a conventional grating acts for a light wave. This effect is referred to as Bragg diffraction. Bragg diffraction was originally demonstrated by deflecting thermal atomic beams [3, 4], and the technique has seen much use in atom-beam interferometers [5]. More recently, similar effects have been achieved using nearly stationary atoms produced using laser cooling or Bose-Einstein condensation [6, 7]. This too has been applied to atom interferometers [8, 9, 10, 11, 12] and other experiments [13, 14, 15].

Although several studies of Bragg diffraction were performed using atomic beams [3, 4, 5], the large spread in atomic velocity complicated comparisons with theory and also limited the fidelity with which operations such as beam-splitting and reflection could be applied. Accurate operations are advantageous for most applications. For instance, in nearly all interferometer schemes, imperfect operations will reduce signal visibility through the loss of atoms and the presence of non-interfering atoms in the measured output states. More specifically, for interferometers such as in [9, 11], a stationary condensate is split into two packets moving apart. After some time the packets are brought back together and returned to zero velocity. Any residual atoms left at zero velocity after the splitting operation would interfere with the final recombination and introduce phase errors.

With ultracold atoms, quite accurate Bragg manipulations can be achieved. This has been observed in experiments for some time, but a quantitative study has not been performed. In this paper, we investigate the dependence of the operations on two key parameters, the laser intensity and residual atomic velocity, and find good agreement with theoretical expectations. We also report a level of fidelity that has not, to our knowledge, been

previously seen. This demonstrates the possibility to control atomic motion with a degree of precision comparable to that previously achievable only with internal state transitions.

We focus attention on the beam-splitting operation first implemented in [9] and the reflection operation demonstrated in [11]. A general understanding of these manipulations can be obtained by considering the three states $|0\rangle$, $|+v_0\rangle$ and $|-v_0\rangle$, where the state labels give the atomic velocity and $v_0 = 2\hbar k/M$ for atomic mass M and light wavenumber k . As will be detailed below, the optical standing wave potential can be expressed as $\hbar\beta \cos 2ky$. This couples the state $|0\rangle$ to the states $|\pm v_0\rangle$ via matrix elements $\hbar\beta/2$, driving the beam-splitting transition

$$|0\rangle \leftrightarrow |+\rangle \equiv \frac{1}{\sqrt{2}}(|v_0\rangle + |-v_0\rangle). \quad (1)$$

However, since the $|0\rangle$ and $|+\rangle$ states have energies that differ by $mv_0^2/2$ the transition is not resonant and cannot be made with perfect efficiency using a single pulse. An appropriate pulse can, however, create the superposition state $(|0\rangle + |+\rangle)/\sqrt{2}$. Due to the energy difference, the phase of this superposition changes as the state evolves freely in time, eventually becoming $(|0\rangle - |+\rangle)/\sqrt{2}$. If a second identical pulse is then applied, the state evolves to the desired $|+\rangle$. This analysis follows that of Wu *et al.* [16].

The reflection operation $|+v_0\rangle \leftrightarrow |-v_0\rangle$ couples two states of equal energy, so multiple pulses are not required. However, the coupling between them is only second order, making the transition rather slow. Efficient operation can be obtained by noting that the state $|+v_0\rangle$ can be expressed as $(|+\rangle + |-\rangle)/\sqrt{2}$, where the antisymmetric state $|-\rangle = (|+v_0\rangle - |-v_0\rangle)/\sqrt{2}$ has no coupling to the $|0\rangle$ state. The $|-\rangle$ state does, however, acquire an energy shift while the Bragg beam is on, making its phase evolve in time. For the appropriate pulse intensity and duration, the $|+\rangle$ branch of the wave function makes two full Rabi oscillations between $|0\rangle$ and $|+\rangle$, ending up back in $|+\rangle$ as it started. At the same time, the $|-\rangle$ branch acquires a π phase shift, making the total state $(|+\rangle - |-\rangle)/\sqrt{2} = |-v_0\rangle$ and achieving the desired reflection.

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To study these operations, we applied them to Bose-Einstein condensates consisting of about 10^4 ^{87}Rb atoms held in a magnetic guide. The apparatus has been described previously [17]. One feature of the guide is its relatively weak confinement: for the experiments described here, the harmonic oscillation frequencies were $\omega_x = 2\pi \times 7.4$ Hz, $\omega_y = 2\pi \times 0.8$ Hz, and $\omega_z = 2\pi \times 4.3$ Hz, with the Bragg laser beam parallel to the y -axis. As a result, the atomic densities are relatively low and interaction effects are negligible.

Another relevant aspect of the apparatus is the procedure by which atoms are loaded into the guide. The condensates are produced in a much tighter trap suitable for evaporative cooling. The tight trap and the guide are coincident, and the guide is loaded by slowly ramping up the guide field and then ramping down the tight trap field. During the final ramp, the trap frequencies pass through a value of 60 Hz, at which point ambient fields with a few mG amplitude can excite center-of-mass oscillation of atoms. Velocities of up to 0.5 mm/s were observed as a result, which is large enough to affect the splitting and reflection operations. Removing nearby noise sources reduced the velocity amplitude to about 0.2 mm/s. This was further controlled by synchronizing the loading process with the power line frequency and adjusting the start of the experiment to occur at a turning point of the atomic motion where the velocity was near zero.

Alternatively, we could use the loading process to controllably impart a velocity to the atoms. In this case, the centers of the tight trap and the guide were deliberately offset, and we suddenly turned off the tight trap current before it reached zero. This caused the atoms to oscillate with a large amplitude. By starting the experiment at different points in the cycle, various atomic velocities were sampled and the effect of velocity on the Bragg operations could be investigated. Since the Bragg operations require less than 1 ms to perform, the atomic velocity was essentially constant during the experiment.

The Bragg standing wave is produced by a home-built diode laser. The laser is tuned 12.8 GHz blue of the $5S_{1/2}$ to $5P_{3/2}$ laser cooling transition, at a wavelength of 780.220 nm. The frequency is stabilized using a cavity-transfer lock [18] referenced to a resonant laser locked via saturated absorption. An acousto-optic modulator (AOM) is used to switch the beam on and off, and also to adjust the beam intensity. The output of the AOM is coupled into a single-mode optical fiber that provides spatial filtering and pointing stabilization. The fiber output has a power of up to 12 mW and an approximately Gaussian profile with beam waist of 0.7 mm. The standing wave is generated by passing the beam through the vacuum cell and retro-reflecting it using an external mirror. The cell was constructed with vacuum windows that were anti-reflection coated on both sides. We found this to be critical: The cell used in Ref. [11] had uncoated Pyrex windows, and the multiple reflections produced a speckle pattern in the reflected beam with intensity vari-

ations of over 50%. This variation made it difficult to achieve consistent results.

The experiments were performed by applying one or more pulses of the Bragg beam and then allowing the atoms to evolve freely for 30 ms so that packets with different velocities would separate in space. The atoms were observed by passing a resonant probe beam through them into a camera, with the resulting images showing the positions of the packets and the number of atoms in each. The images were analyzed by fitting the packets to Gaussian profiles. For faint packets, the widths of the profiles were fixed to match those observed for packets containing many atoms, typically 10 μm and 70 μm in the x and y directions, respectively.

In general, several packets are observed in a given image. The packets are labeled according to their velocity index n , defined by $v = v_i + nv_0$ where v is the observed velocity and v_i the initial velocity. The fraction of atoms in each packet is denoted N_n . The accuracy of the operation is quantified using the fidelity, defined as

$$F = |\langle \psi_0 | \psi \rangle|^2 \quad (2)$$

for desired state ψ_0 and observed state ψ . For instance, an ideal splitting operation produces the state $|+\rangle$, so if the actual state is

$$|\psi\rangle = \sum_n c_n |v_i + nv_0\rangle \quad (3)$$

then the fidelity would be

$$F_{\text{split}} = \frac{1}{2} |c_{+1} + c_{-1}|^2 = \frac{1}{2} (N_{+1} + N_{-1} + 2\text{Re } c_{+1}^* c_{-1}). \quad (4)$$

In general this depends on the phase difference between c_{+1} and c_{-1} , which of course cannot be determined by simple imaging. Moreover, interferometer operation does not depend on this phase either as long as it is constant, since any phase difference imposed by the splitting operation can be absorbed into the definitions of the states $|\pm v_0\rangle$ without changing the ultimate results. For this reason, we ignore any phase effects and estimate the fidelity as

$$F_{\text{split}} = \frac{1}{2} (N_{+1} + N_{-1} + 2\sqrt{N_{+1}N_{-1}}). \quad (5)$$

We observe the most accurate splitting operation when using zero-velocity atoms and a Bragg beam power of 0.34 mW. Figure 1 shows the results obtained as the power and velocity were varied around these values. Note that the velocity width observed here corresponds to a temperature of 60 nK, which illustrates the benefit of performing such experiments using condensates. Each data point shown represents a single measurement. Repeated measurements under the same conditions showed variations of about 0.01 in N_0/N . At the optimal parameters, the best images yielded fidelities of about 0.995, with repeated measurements consistently above 0.99. Image noise limits the accuracy of our analysis

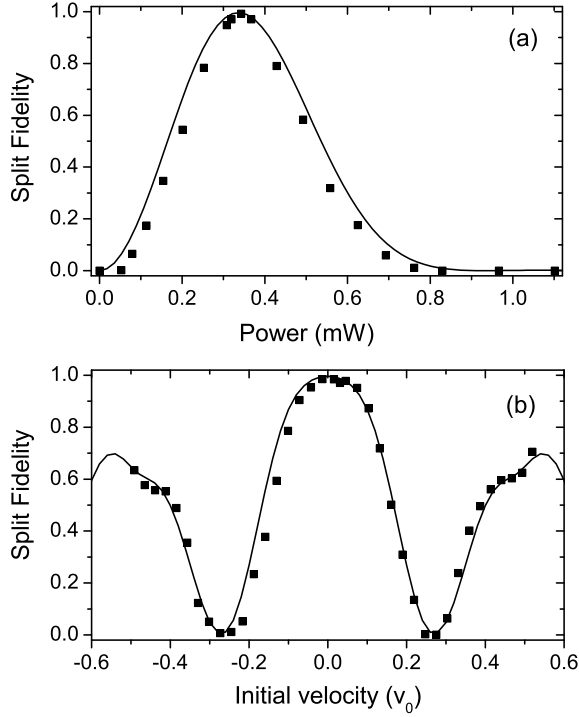


FIG. 1: Fidelity of splitting operation. Data points show the experimentally measured fidelity as a function of (a) the Bragg laser power and (b) initial atomic velocity. The solid curves are theoretical calculations and include a calibration adjustment of 1.19 to relate the standing wave intensity to the measured power. The pulse sequence consisted of a pulse with duration $24 \mu\text{s}$, a $33 \mu\text{s}$ delay, and a second pulse identical to the first.

to about this level, but we also typically observe longer-term variations that we attribute to experimental variations in the atomic velocity, beam alignment, and laser power. Nonetheless it was possible to maintain split fidelities near 0.99 over a period of several hours.

Similar results for the reflection operation are shown in Fig. 2. Here the fidelity is given by N_{+2} , since the initial velocity is close to $-v_0$. The optimal fidelity was about 0.94.

The theoretical curves shown in Figs. 1 and 2 are calculated by solving the Schrödinger equation in the presence of the optical potential,

$$i\frac{d\psi}{dt} = \left[-\frac{\hbar}{2M} \frac{\partial^2 \psi}{\partial y^2} + \beta \cos(2ky)\psi \right], \quad (6)$$

using the Bloch expansion

$$\psi(y, t) = \sum_n c_n(t) e^{i(2nk + \kappa)y}, \quad (7)$$

where $\kappa = Mv_i/\hbar$ [16]. The coefficients c_n then satisfy

$$i\frac{dc_n}{dt} = \frac{\hbar}{2M} (2nk + \kappa)^2 c_n + \frac{\beta}{2} (c_{n-1} + c_{n+1}). \quad (8)$$

We solved equations (8) numerically, including indices n up to ± 4 . (Negligible population was observed in the

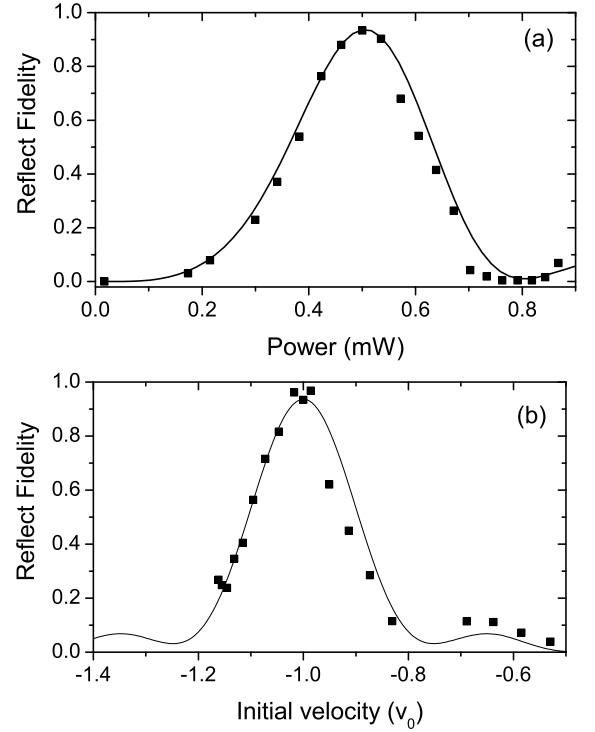


FIG. 2: Fidelity of reflection operation. Data points show the measured values and curves are a theoretical calculation. The operation consists of a single pulse with duration $76 \mu\text{s}$.

$c_{\pm 4}$ states.) We applied the initial condition $c_0 = 1$ and calculated the fidelity as described above. The results show good agreement with the experimental data, but to achieve this it was necessary to calibrate the standing wave amplitude β .

To calculate β , we treat the atoms as two-level systems, in which the $5S_{1/2} F = 2$ ground hyperfine state is coupled to the $5P_{3/2}$ excited state. We neglect the excited state hyperfine splitting, since it is small compared to the laser detuning Δ . The optical potential is then given by [1]

$$V(y) = \frac{\hbar\Gamma^2}{8\Delta} \frac{I(y)}{I_{\text{sat}}} \quad (9)$$

where $\Gamma = 3.8 \times 10^7 \text{ s}^{-1}$ is the excited state linewidth, I is laser intensity, and $I_{\text{sat}} = 2.5 \text{ mW/cm}^2$ is the saturation intensity for linearly polarized light. For an incident beam with power P and Gaussian waist w , the standing wave intensity is given by

$$I(y) = \frac{8P}{\pi w^2} \cos^2(ky) = \text{const} + \frac{4P}{\pi w^2} \cos 2ky. \quad (10)$$

The constant term has no effect, so comparison with the form $V = \hbar\beta \cos(2ky)$ gives

$$\beta = \frac{\Gamma^2 P}{2\pi\Delta I_s w^2} \approx 10\hbar\omega_r \times \frac{P}{\text{mW}} \quad (11)$$

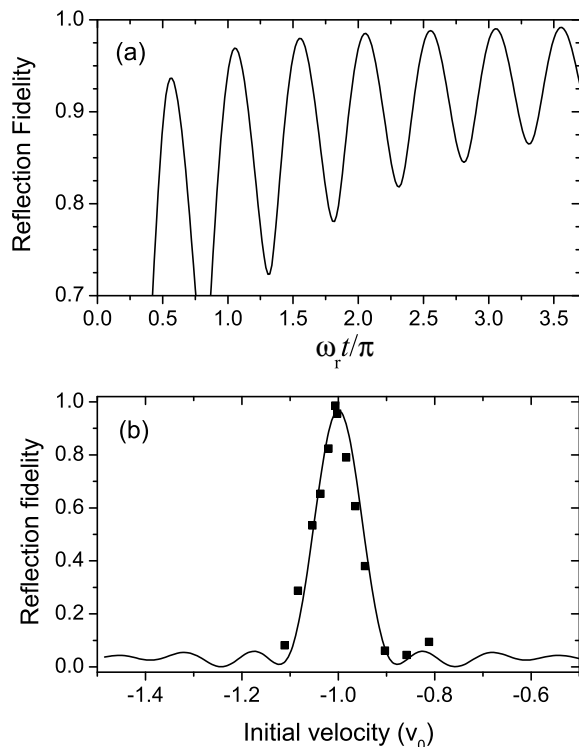


FIG. 3: (a) Optimized reflection fidelity vs. pulse duration. For each pulse duration t , the standing wave amplitude β was adjusted to provide the best reflection fidelity for atoms traveling at $v_i = v_0$. The results of Fig. 2 correspond to the peak near $\omega_r t = \pi/2$. (b) Experimental results for a longer pulse. Here $\omega_r t = 3.3$ and the beam power was 0.37 mW.

where the recoil frequency ω_r is defined as $\hbar k^2/2M \approx 2.36 \times 10^4 \text{ s}^{-1}$. The optimum theoretical fidelity for the split operation occurs at $\beta = 2.8\omega_r$, or $P = 0.28 \text{ mW}$, about 20% different from the observed peak. We attribute this discrepancy to experimental errors in the intensity calibration. In particular, intensity variations of this order can result from small misalignments of the beam on the atoms or deviations of the beam profile from a Gaussian. We therefore include a calibration factor in the theory curve, and adjust it to match the measured peak. The ratio between the powers for the splitting and reflection operations was about 0.65, in good agreement with theoretical expectation $\beta_{\text{reflect}} = 4.4\omega_r$.

It is noteworthy that the optimum reflection fidelity is lower than that achievable for the splitting operation. One solution is to use a longer pulse with lower intensity [4]. In the limit of a very long, very weak pulse, perfect reflection should be possible. We show how this regime can be approached in Fig. 3(a). This curve was calculated by varying the pulse length and at each point numerically optimizing the value of β . It is observed that

the reflection fidelity does indeed improve, though not monotonically. This behavior can be understood with reference to the three-level model discussed previously. If the problem is solved in detail, it is found that the initial state $|+v_0\rangle = (|-\rangle + |+\rangle)/\sqrt{2}$ evolves to

$$|\psi\rangle = \frac{e^{-2i\omega_r t}}{\sqrt{2}} \left[e^{-2i\omega_r t} |-\rangle - i \frac{\beta\sqrt{2}}{X} \sin \frac{Xt}{2} |0\rangle + \left(\cos \frac{Xt}{2} - i \frac{4\omega_r}{X} \sin \frac{Xt}{2} \right) |+\rangle \right] \quad (12)$$

where $X = (2\beta^2 + 16\omega_r^2)^{1/2}$. The desired state $| -v_0 \rangle = (|+\rangle - |-\rangle)/\sqrt{2}$ is achieved when $2\omega_r t = n\pi$ and $Xt/2 = (n+1)\pi$ for integer $n > 0$. The peaks observed in Fig. 3 indeed occur at $\omega_r t \approx n\pi/2$. Solving for the required intensity yields $\beta = 2\omega_r(4n+2)^{1/2}/n$. The reflection operation is more accurate for large n because coupling to higher velocity states is reduced as β decreases.

We experimentally observed the second maximum, at pulse time $t = 140 \mu\text{s}$ and $\beta = 3.0\omega_r$. The results are shown in Fig. 3(b), where a peak fidelity of about 0.98 is observed. The longer pulse duration, however, results in greater sensitivity to the atom velocity. In situations where the velocity cannot be perfectly controlled, the shorter pulse may be preferable.

Reference [16] also points out that the double-pulse splitting sequence can be used efficiently for higher order operations, driving for instance $|0\rangle \rightarrow (|+2v_0\rangle + |-2v_0\rangle)/\sqrt{2}$. We implemented these operations as well. For the order-2 split above, we observed a peak fidelity of about 0.92, while the order-3 split produced atoms with velocity $\pm 3v_0$ at a fidelity of 0.8. Theory predicts fidelities of 0.99 and 0.97 respectively. For at least the third-order result, the relatively poor experimental performance can be attributed to the high sensitivity of the operation to the pulse intensity and duration. Our timing resolution is limited by the 100-ns switching time of the AOM, and we observed the order-3 pulse to be sensitive on that time scale. We did not implement an order-4 split because it requires greater intensity than we had available.

In summary, we have investigated the optical manipulation of atomic wave packets. We demonstrated splitting and reflecting with high fidelity and good agreement with theoretical expectations. We hope that these results will prove useful for the development of atom interferometers and other applications of atom optics.

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